LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2013**

MT 5505/MT 5501 - REAL ANALYSIS

Date: 05/11/2013 Dept. No. Max. : 100 Marks

Time: 9:00 - 12:00

PART – A

Answer **ALL** questions: (10 X 2 = 20)

1. State the principle of induction.
2. Differentiate countable and uncountable sets.
3. Define open cover.
4. Define adherent point.
5. Define a convergent sequence.
6. Define complete metric space and give an example of a space which is not complete.
7. Define open ball and closure of a set E.
8. Define local maximum and local minimum of a function at a point.
9. Define limit superior and limit inferior of a real sequence.
10. Define telescopic series.

PART – B

Answer any **FIVE** questions: (5 X 8 = 40)

11. State and prove Cauchy-schwarz inequality.
12. Prove that the set of all real numbers is uncountable.
13. State and prove the representation theorem for open sets on the real line.
14. Prove that every Cauchy sequence is convergent in the Euclidean space R^k.
15. Prove that the continuous image of a compact set is compact.
16. State and prove Rolle's theorem.
17. State and prove the linearity property of the Riemann Stieltjes integral.
18. State and prove the formula for Integration by parts.

PART – C

Answer any **TWO** questions: (2 X 20 = 40)

19. Prove that if F is a countable collection of countable sets, then the union of all sets in F is also countable.
20. State and prove the Bolzano Weierstrass theorem.
21. Let f be a strictly increasing and continuous on a compact interval [a, b]. Then prove that f^{1} is continuous and strictly increasing on the interval [f(a), f(b)].
22. a) State and prove Taylor's formula with remainder.

    b) Assume $f \in R(\alpha)$ on [a, b]. Then prove that the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists and $\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.

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